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# Dynamics of pairwise entanglement between two Tavis–Cummings atoms

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**Abstract**

We investigate the time evolution of pairwise entanglement between two Tavis–Cummings atoms for various entangled initial states, including pure and mixed states. We find that the phenomenon of entanglement sudden death behaviors is distinct in the evolution of entanglement for different initial states. What deserves mentioning here is that the initial portion of the excited state in the initial state is responsible for the sudden death of entanglement, and the degree of this effect also depends on the initial states.

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(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Entanglement, one of the most striking features of quantum mechanics, has been recognized as an important resource for quantum information processing [1]. It can be exploited to accomplish quantum teleportation [2], superdense coding [3], quantum cryptographic key distribution [4] and so on. However, entanglement in a quantum system may deteriorate due to interaction with background noise or with other systems usually called environments. As a result, it is one of the most challenging to store entanglement in two spatially separated atoms for a sufficiently long time. So in order to realize various quantum information processing schemes, a deep understanding of this entanglement decoherence is desirable.

In recent years, there has been an ongoing effort applied to study the dynamics of entanglement since the work of Yu and Eberly [5], in which the entanglement between the two particles coupled with two independent environments became completely vanishing in a finite time, termed entanglement sudden death (ESD). This surprising phenomenon, contrary to intuition on the decoherence also appears in many other scenarios [6–14]. However, it is still unclear what reason causes this phenomenon and what is the physics behind though the extensive progress in understanding the disentanglement. Recently, [15] revealed that the dynamics of entanglement and its revival in the Tavis–Cummings model origin from the

  

transfer between the system and its surroundings, but the relation between the ESD and the transfer of energy is not clear. In this paper, we still concern with the dynamics of entanglement between two Tavis–Cummings atoms for various initial entangled states, including pure and mixed states. Different from the previous work, we focus on the relation between the ESD and different mixed portions in the initial states. Our results will be helpful in understanding the origin of the ESD.

This paper is organized as follows. In section 2, we first review the description of the Tavis–Cummings model. In section 3, the dynamic evolution of the entanglement between the two Tavis–Cummings atoms for different pure and mixed initial states is discussed in detail respectively, and we conclude in section 4.

## 2. Description of the model

We consider two identical two-level atoms A and B coupled with a single-model cavity field, and there are no interactions between the two atoms, the Hamiltonian describing the system is [16] ($\hbar = 1$),

$$H = \frac{\omega_0}{2}\left(\sigma_A^z + \sigma_B^z\right) + \omega a^+ a + g \sum_{i=A,B} \left(a\sigma_i^+ + a^+\sigma_i^-\right), \tag{1}$$

where $\sigma_z$ is the atomic inversion operator, $a$ ($a^+$) denotes the annihilation (creation) operator for the single-model cavity field, and $\sigma_i^+ = |e\rangle_i\langle g|$ ($\sigma_i^- = |g\rangle_i\langle e|$) is the atomic rising (lowering) operator. $\omega_0$ and $\omega$ are the frequencies of atomic transition and cavity, respectively, and $g$ represents the strength of atoms and cavity. The ground and excited states for the atom $i$ ($i$=$A$, $B$) are denoted by $|g\rangle_i$ and $|e\rangle_i$. Here by assuming that the atoms A and B resonantly couple with the single-model cavity field, under the rotating wave approximation, the Hamiltonian (1) in the interaction picture turns out to be

$$H_I = g \sum_{i=A,B} (a\sigma_i^+ + a^+\sigma_i^-). \tag{2}$$

Then using Hamiltonian (2), the time evolution operator $U(t) = \exp(-\mathrm{i}Ht)$ can be calculated in the atomic basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$ as the form [17]

$$U(t) = \begin{pmatrix} \mu + 1 & -\mathrm{i}g\nu & -\mathrm{i}g\nu & \chi \\ -\mathrm{i}g\nu^+ & \frac{1}{2}(\cos\Omega t + 1) & \frac{1}{2}(\cos\Omega t - 1) & -\mathrm{i}g\eta \\ -\mathrm{i}g\nu^+ & \frac{1}{2}(\cos\Omega t - 1) & \frac{1}{2}(\cos\Omega t + 1) & -\mathrm{i}g\eta \\ \chi^+ & -\mathrm{i}g\eta^+ & -\mathrm{i}g\eta^+ & \mu + 1 \end{pmatrix}, \tag{3}$$

where

$$\begin{aligned} &\mu = 2g^2 a(\Gamma - \Theta)a^+, \qquad \chi = 2g^2 a(\Gamma - \Theta)a, \qquad \nu = aS, \qquad \eta = Sa, \\ &\Gamma = \Theta\cos\Omega t, \qquad S = \Omega^{-1}\sin\Omega t, \end{aligned} \tag{4}$$

with $\Omega^2 = \Theta^{-1} = 2g^2(2a^+a + 1)$.

## 3. The entanglement between the two atoms

When the two two-level atoms A and B interact with the single-model cavity field C, the density operator for the total system at time $t$ is

$$\rho(t) = U(t)(\rho_{AB}(0) \otimes \rho_C(0))U^+(t). \tag{5}$$

After performing a partial trace over the cavity C, the reduced density matrix for the atomic system can be expressed as $\rho_{AB}(t) = Tr_C\rho(t)$. Previous papers [6, 7] have demonstrated that the dynamical behavior of the entanglement between two atoms depends on the initial state deeply. Therefore, in what follows, we will concentrate on the entanglement evolution for different initial states, including pure and mixed initial states.

### *3.1. Entanglement evolution for an arbitrary pure initial state*

In this section, we will take a combination of the two types of Bell states as our initial states for each of the two atoms, namely

$$\rho_{AB}(0) = |\Psi_{AB}\rangle\langle\Psi_{AB}|, \tag{6}$$

and

$$\rho_{AB}(0) = |\Phi_{AB}\rangle\langle\Phi_{AB}|, \tag{7}$$

where

$$|\Psi_{AB}\rangle = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{AB}, \qquad |\Phi_{AB}\rangle = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{AB}. \tag{8}$$

First, we choose (6) as the initial atomic state and the cavity is prepared initially in the Fock state $|n\rangle$. The initial state for the total system (1) is given by

$$\rho(0) = |\Psi_{AB}\rangle\langle\Psi_{AB}| \otimes |n\rangle_C\langle n|. \tag{9}$$

Then the reduced density matrix for the atomic system $\rho_{AB}(t) = Tr_C[U(t)\rho(0)U^+(t)]$ can be written in the basis $\{|ee\rangle = |1\rangle, |eg\rangle = |2\rangle, |ge\rangle = |3\rangle, |gg\rangle = |4\rangle\}$ as

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{10}$$

with

$$\begin{aligned} \rho_{11} &= \frac{n}{4n+2}\sin^2\theta(1+\sin 2\alpha), \\ \rho_{22} = \rho_{33} &= \frac{1}{4}(\cos^2\theta - 2\cos\theta\cos 2\alpha - \sin^2\theta\sin 2\alpha + 1), \\ \rho_{23} = \rho_{32} &= \frac{1}{4}[\sin 2\alpha(1+\cos^2\theta) - \sin^2\theta], \\ \rho_{44} &= \frac{n+1}{4n+2}\sin^2\theta(1+\sin 2\alpha), \end{aligned} \tag{11}$$

where $\theta = \sqrt{4n+2}gt$. It is proved that the concurrence [18] is a useful entanglement measure for a pair of qubits. Let $\rho_{12}$ be the density matrix of two qubits and it can denote either a pure or a mixed state. Then the concurrence is calculated as

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{12}$$

where the quantities $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4$ are the square roots of the eigenvalues of the matrix $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, $\sigma_y$ is the normal Pauli operator. The concurrence $C = 0$ corresponds to an unentangled state and $C = 1$ for a maximally entangled state. Therefore,

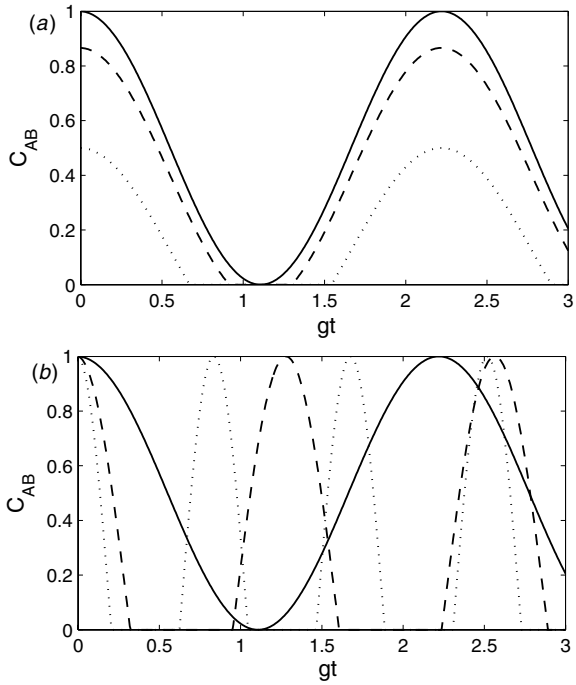


**Figure 1.** The time evolution of the concurrence $C_{AB}$ when the atoms are initially in $|\Psi_{AB}(0)\rangle = (\cos\alpha|eg\rangle + \sin\alpha|ge\rangle)_{AB}$, (*a*) for $n = 0$ and $\alpha = \frac{\pi}{4}$ (solid line), $\alpha = \frac{\pi}{3}$ (dashed line), $\alpha = \frac{\pi}{12}$ (dotted line); (*b*) for $\alpha = \frac{\pi}{4}$ and $n = 0$ (solid line), $n = 1$ (dashed line), $n = 3$ (dotted line),

from equation (11), the concurrence $C(\rho_{AB})$ for the two atoms is obtained as

$$C_{AB} = \max\{0, \rho_{23} - \sqrt{\rho_{11}\rho_{44}}\}$$
$$= \max\left\{0, \frac{\sin 2\alpha(1+\cos^2\theta) - \sin^2\theta}{4} - \frac{\sin^2\theta(1+\sin 2\alpha)}{4n+2}\sqrt{n(n+1)}\right\}. \tag{13}$$

When the cavity lies in a vacuum state initially, i.e., $n = 0$, equation (13) becomes

$$C_{AB} = \tfrac{1}{4}\max\{0, \sin 2\alpha(1+\cos^2\sqrt{2}gt) - \sin^2\sqrt{2}gt\}. \tag{14}$$

It is easy to see that the entanglement is symmetrical with respect to the plane of $\alpha = \frac{\pi}{2}$ and the entanglement may occur only when $0 < \alpha < \pi/2$. For example, there is no entanglement when $\alpha = \frac{3\pi}{4}$, though the initial atomic state reduces to one of the Bell states $\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$ at the moment. In figure 1(*a*) we plot the time evolution of the concurrence $C_{AB}$ for three different values of the superposition parameter $\alpha$ when $n = 0$. For $\alpha = \frac{\pi}{4}$, we can see that the entanglement can survive for an infinite time periodically. However, for $\alpha = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{12}$, entanglement goes through a sudden death and will remain zero for a period of time before entanglement recovers, and the time regions of disentanglement is shorter for $\alpha = \frac{\pi}{3}$ than that of the case for $\alpha = \frac{\pi}{12}$. So the length of the time for zero entanglement is dependent on the degree of entanglement of the initial state, and the smaller the degree of the entanglement in the initial state, the longer the state will stay in the disentangled state. Figure 1(*b*) shows that the effect of photon number on time evolution of the concurrence $C_{AB}$ when the atoms are initially in the maximally entanglement state $|\Psi_{AB}(0)\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$. We find that

the phenomenon of ESD can be induced by introducing the photon number, and the more the photon number, the shorter the length of the time for zero entanglement.

Consider another type of pure initial atomic state (7), the reduced density matrix $\rho_{AB}$ has the form

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{15}$$

with

$$\begin{aligned} \rho_{11} &= \left(\frac{n+1}{2n+3}\cos\theta_1 + \frac{n+2}{2n+3}\right)^2 \cos^2\alpha, \\ \rho_{22} &= \frac{n+1}{4n+6}\sin\theta_1^2\cos^2\alpha + \frac{n}{4n-2}\sin\theta_2\sin^2\alpha, \\ \rho_{44} &= \frac{(n+1)(n+2)}{(2n+3)^2}(\cos\theta_1 - 1)^2\cos^2\alpha + \frac{(n\cos\theta_2 + n - 1)^2}{(2n-1)^2}\sin^2\alpha, \\ \rho_{14} &= \frac{\sin 2\alpha(n\cos\theta_2 + n - 1)}{2(2n+3)(2n-1)}[(n+1)\cos\theta_1 + n + 2], \\ \rho_{23} &= \rho_{32} = \rho_{33} = \rho_{22}, \qquad \rho_{41} = \rho_{14}, \end{aligned} \tag{16}$$

where $\theta_1 = \sqrt{4n+6}gt$ and $\theta_2 = \sqrt{4n-2}gt$. The concurrence $C_{AB}$ is obtained as

$$C_{AB} = \max\{0, \rho_{14} - \rho_{23}\}. \tag{17}$$

For the case of $n = 0$,

$$C_{AB} = \tfrac{1}{6}\max\{0, \sin 2\alpha[\cos(\sqrt{6}gt) + 2] - \cos^2\alpha\sin^2(\sqrt{6}gt)\}. \tag{18}$$

It is easy to examine that $0 < \alpha < \pi/2$ is also the sufficient condition for entanglement existing in the time evolution, but the entanglement is no longer symmetrical with respect to the plane of $\alpha = \frac{\pi}{2}$. One could find that the ESD appears only for a much smaller $\alpha$ in this case in contrast to that of the initial atomic state (6), as is shown in figure 2(*a*). We can conclude that the ESD effects on entanglement evolution for the initial atomic state (6) are much stronger than that of the initial atomic state (7). But for $n \neq 0$, figure 2(*b*) shows that the ESD appears by increasing photon number and the decoherence induced by the photon number is much more evident for the initial atomic state (7) than that of initial state (6). Thus the dynamical behavior of entanglement between these two Tavis–Cummings atoms depends not only on the degree of entanglement of the initial state but also on the form of the initial state.

### *3.2. Entanglement evolution for the mixed initial states*

In the following, we will investigate the entanglement character considered above when the two atoms are supposed to be initially prepared in a mixed state, such as $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|$ $(0 \leqslant a < 1)$. For simplicity, we assume that the cavity lies in the vacuum state $n = 0$ initially, and then the initial state for the total system becomes $\rho(0) = [(1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|] \otimes |0\rangle_C\langle 0|$. After some calculation, the entanglement for $\rho_{AB}(t) = Tr_C[U(t)\rho(0)U^+(t)]$ can be given easily, and the form of $\rho_{AB}(t)$ is

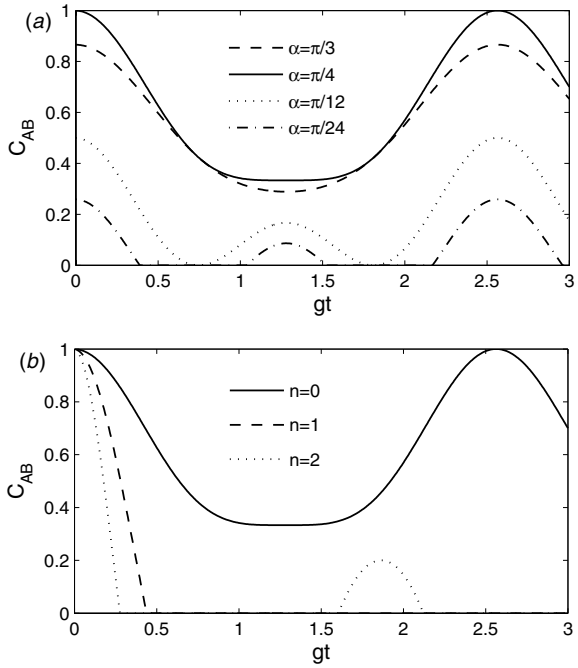


**Figure 2.** The time evolution of the concurrence $C_{AB}$ when the atoms are initially in the $|\Phi_{AB}(0)\rangle = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)_{AB}$, (*a*) for $n = 0$ and different values of $\alpha$; (*b*) for $\alpha = \frac{\pi}{4}$ and different photon number $n$.

the same as (10) with

$$\rho_{11} = \frac{a}{9}(\cos\sqrt{6}gt + 2)^2,$$
$$\rho_{44} = \frac{2a}{9}(\cos\sqrt{6}gt - 1)^2 + \frac{1-a}{2}\sin^2\sqrt{2}gt(1 + \sin 2\alpha), \tag{19}$$
$$\rho_{23} = \rho_{32} = \frac{1-a}{4}\sin 2\alpha(1 + \cos^2\sqrt{2}gt) - \frac{1-a}{4}\sin^2\sqrt{2}gt + \frac{a}{6}\sin^2\sqrt{6}gt.$$

We then obtain the concurrence by $C_{AB} = \max\{0, \rho_{23} - \sqrt{\rho_{11}\rho_{44}}\}$. In figure 3, the time evolution of the concurrence $C_{AB}$ for different values of parameter $a$ is plotted when the two atoms initially in the maximally entangled state $|\Psi_{AB}\rangle$ for $\alpha = \frac{\pi}{4}$ with the double excited state $|ee\rangle$ mixed. In contrast to the case $a = 0$ discussed above where no ESD occurs, we find that the ESD always happens no matter how small portion ($a$) of the double excitation $|ee\rangle$ is in the initial state, and the length of the time for zero entanglement is increased as the portion of the double excitation becomes large. In the previous section, we have proved that the ESD effects depend on the degree of entanglement of the initial state, so it seems that the decrease of the degree of entanglement of the initial state by mixing the excited state $|ee\rangle$ results in the appearance of ESD.

Now we consider the entangled state $|\Psi_{AB}\rangle$ mixed with the double ground state $|gg\rangle$ as the initial atomic state. For the case $n = 0$, $\rho(0) = [(1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|gg\rangle_{AB}\langle gg|] \otimes |0\rangle_C\langle 0|$

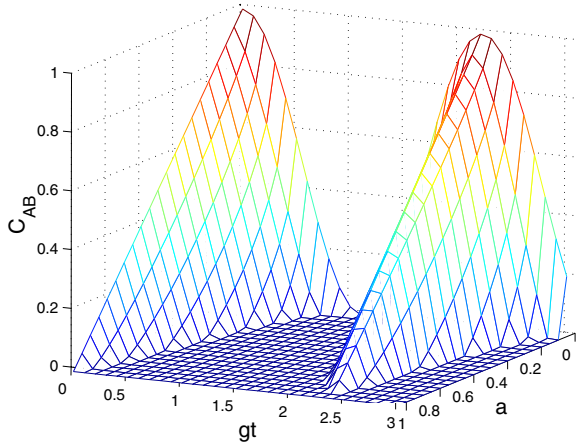


**Figure 3.** The time evolution of the concurrence $C_{AB}$ when the atoms are initially in $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|$ for $\alpha = \pi/4$.

$(0 \leqslant a < 1)$. For this, $\rho_{AB}(t)$ has the form as (10), with

$$\begin{aligned}\rho_{11} &= 0,\\ \rho_{44} &= \frac{1-a}{2}\sqrt{2}gt(1+\sin 2\alpha) + a,\\ \rho_{23} = \rho_{32} &= \frac{1-a}{4}[\sin 2\alpha(1+\cos^2\sqrt{2}gt) - \sin^2\sqrt{2}gt],\end{aligned} \tag{20}$$

and we can obtain the concurrence

$$\begin{aligned}C_{AB} &= \max\{0, \rho_{23}\}\\ &= \frac{1-a}{4}\max\{0, \sin 2\alpha(1+\cos^2\sqrt{2}gt) - \sin^2\sqrt{2}gt\}.\end{aligned} \tag{21}$$

Comparing equation (21) with equation (14), we find that the time evolution of the entanglement in this type of mixed initial state is like that of the initial pure state (9), except that the values of the concurrence is decreased by $1-a$ for a fixed time $t$ with increasing the portion of the double ground states $|gg\rangle$ in the initial state, but the concurrence will not vanish for $a < 1$. Namely, the ESD effects are independent of the portion of the double ground states $|gg\rangle$ in the initial state, which is different from that of the double excitation $|ee\rangle$, though mixing $|gg\rangle$ also depresses the entanglement of the initial state. So all that causes the decrease of the entanglement of the initial state cannot lead to the ESD.

Next we consider the initial state $\rho(0) = [(1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|eg\rangle_{AB}\langle eg|] \otimes |0\rangle_C\langle 0|$ $(0 \leqslant a < 1)$. Skipping the details, we give the concurrence directly

$$C_{AB} = \tfrac{1}{4}\max\{0, (1-a)\sin 2\alpha(1+\cos^2\sqrt{2}gt) - \sin^2\sqrt{2}gt\}, \tag{22}$$

and for the initial state $\rho(0) = [(1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ge\rangle_{AB}\langle ge|] \otimes |0\rangle_C\langle 0|$ $(0 \leqslant a < 1)$, we can obtain the same result as equation (22). Figure 4 gives a plot of time evolution of concurrence $C_{AB}$ for different initial states when $\alpha = \frac{\pi}{4}$ and $a = 0.5$. It suggests that the ESD effects should be induced by mixing the double excited state $|ee\rangle$ or the ground-excited state $|eg\rangle$ with the maximally entangled initial state, but not for the double ground state $|gg\rangle$. In addition, it is interesting to note that the death time for the initial state $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|$ is longer than that of the initial state

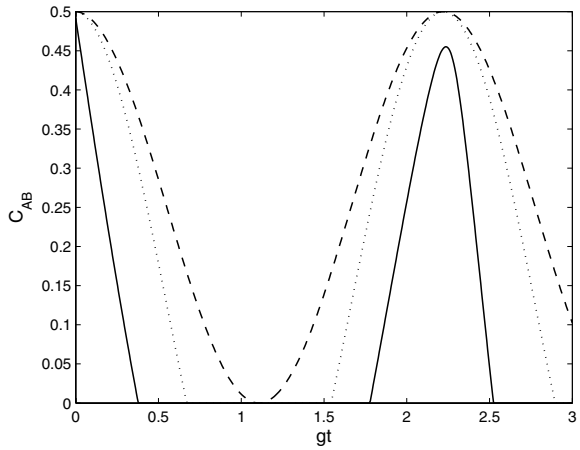


**Figure 4.** The time evolution of the concurrence $C_{AB}$ for different initial atomic states when $a = 0.5, \alpha = \frac{\pi}{4}$. The solid line is for the state $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|$, the dashed line is for the $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|gg\rangle_{AB}\langle gg|$, the dotted line is for the $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|eg\rangle_{AB}\langle eg|$.

$\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|eg\rangle_{AB}\langle eg|$, and this phenomenon is more evident as the value of $a$ is increased. Therefore, we can conclude that the portion of the excited state component in the initial entangled state is responsible for the ESD and the effects of double-excited $|ee\rangle$ are stronger than that of the ground-excited state $|eg\rangle$ or $|ge\rangle$.

The above discussion is about the pure state $|\Psi_{AB}\rangle$ mixed with other states as the initial state. From now on, we move to consider the initial state $\rho(0) = [(1-a)|\Phi_{AB}\rangle\langle\Phi_{AB}| + a|ee\rangle_{AB}\langle ee|] \otimes |0\rangle_C\langle 0|$. Following the method of the previous discussion, we find that $\rho_{AB}(t)$ has the form mentioned in equation (15) with

$$\begin{aligned} \rho_{23} &= \rho_{32} = \rho_{33} = \rho_{22} = \frac{1}{6}\sin^2\sqrt{6}gt(\cos^2\alpha + a\sin^2\alpha), \\ \rho_{14} &= \rho_{41} = \frac{1-a}{6}\sin 2\alpha(2+\cos\sqrt{6}gt), \end{aligned} \tag{23}$$

for which the concurrence is

$$\begin{aligned} C_{AB} &= \max\{0, \rho_{14} - \rho_{23}\} \\ &= \frac{1}{6}\max\{0, (1-a)\sin 2\alpha(2+\cos\sqrt{6}gt) - \sin^2\sqrt{6}gt(\cos^2\alpha + a\sin^2\alpha)\}. \end{aligned} \tag{24}$$

From equation (24), we find that when $\frac{\sqrt{3}}{3} < a < 1$ the ESD always happens, and the death time is prolonged by increasing the value of $a$, which can be seen in figure 5. Coming back to the case of initial state $\rho_{AB}(0) = (1-a)|\Psi_{AB}\rangle\langle\Psi_{AB}| + a|ee\rangle_{AB}\langle ee|$, where the ESD appears no matter how small the amount of additional double excitation component is mixed with the initial state. In this sense, the entangled state $|\Psi_{AB}\rangle$ is more fragile than the entangled state $|\Phi_{AB}\rangle$ against the quantum fluctuation of the initial state.

It can be easily proved that all the conclusions discussed above for the initial state $|\Psi_{AB}\rangle$ mixed with $|gg\rangle$ and $|eg\rangle$ or $|ge\rangle$ can still be valid when the atoms are initially prepared in $|\Phi_{AB}\rangle$ mixed with $|gg\rangle$ and $|eg\rangle$ or $|ge\rangle$, here we will not list in detail.

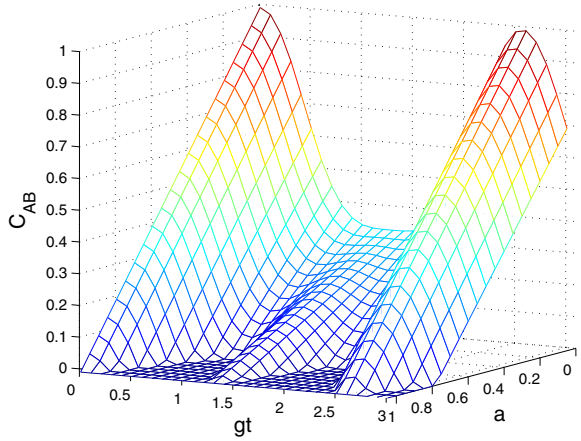


**Figure 5.** The time evolution of the concurrence $C_{AB}$ when the atoms are initially in the $\rho_{AB}(0) = (1-a)|\Phi_{AB}\rangle\langle\Phi_{AB}| + a|ee\rangle_{AB}\langle ee|$ for $\alpha = \pi/4$.

## 4. Conclusion

In conclusion, by the concept of concurrence, we have discussed the time evolution of pairwise entanglement between two Tavis–Cummings atoms for various entangled initial states. For an arbitrary pure initial state, we can make several comments as follows. First, the entanglement sudden death (ESD) appearing in this system depends on the initial state deeply, and this disentanglement effects on time evolution are much stronger for the initial atomic state $|\Psi_{AB}\rangle$ than that for $|\Phi_{AB}\rangle$. Moreover, compared with the vacuum state, we find that a nonzero photon number can induce the phenomenon of ESD, and the death time is also related to the photon number. For the mixed initial state, we show that the initial portion of the excited state is responsible for the sudden death of entanglement, and this effect of double-excited $|ee\rangle$ is stronger than that of the ground-excited state $|eg\rangle$ or $|ge\rangle$. In addition, we also find that the entangled state $|\Psi_{AB}\rangle$ is more fragile than the entangled state $|\Phi_{AB}\rangle$ against the quantum fluctuation of the initial state, because the ESD occurs no matter how small the amount of the additional double excitation component is mixed with $|\Psi_{AB}\rangle$ in the initial state, but for that of $|\Phi_{AB}\rangle$, only when the portion of double excitation exceeds $\frac{\sqrt{3}}{3}$ the ESD can be produced.

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